Guide to Curve Sketching

A Summary of What We Can Learn About the Graph of a Function

Domain and Range

Domain: Are there discontinuities in $f(x)$?

A rational function where the denominator = 0

Radical function (even root) where radicand < 0

Range: What is the maximum and/or minimum of $f(x)$?

Where are the tails pointing? Look at $x \to \infty^{\pm}$. What happens to $f(x)$?

Note that polynomials of even degree have their tails pointing in the same direction so there will be a maximum or minimum for the function.

Other infinities created at asymptotes?

Intercepts

y-intercept: A function has, at most, one. Evaluate: $f(0) = ?$

x-intercept $f(x) = 0$, solve for x.

a) Solve analytically

b) Solve numerically by using graphing calculator or Intermediate Value Theorem to estimate root(s). Then, use Newton's Method to find a precise value.

Continuity

Is the function continuous for all points, c, in the domain? There are three conditions:

a) $f(c)$ must be defined.

b) $\lim_{x \to c} f(x)$ exists.

c) $\lim_{x \to c} f(x) = f(c)$

Vertical Asymptotes

a) Reduce a rational expression to lowest terms. (Common factors between numerator and denominator produce removable, one-point discontinuities.)

a) Set *reduced* denominator = 0 and solve.

Horizontal Asymptotes

$f(x)$ is asymptotic to the horizontal line $y = L$ (where L is some constant, real number) if:

a) $\lim_{x \to \infty^+} f(x) = L$ or b) $\lim_{x \to \infty^-} f(x) = L$

(Note that the limit on the left does not have to equal the limit on the right.)

If the degree of the numerator = 1 + degree denominator, the <u>oblique</u> <u>asymptote</u> is the line

$$q(x) \text{ where } \frac{n(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

Relative Extrema

Relative extrema occur at the function's critical number(s):

End points. (see Domain)

Where $f'(x)$ does not exist.

Values of x where $f'(x) = 0$.

Remember, the extrema are *points* with coordinates $(x, f(x))$.

Concavity

Use the second derivative to determine which way the graph is curving:

$f''(x) > 0$ means the graph is concave up.

$f''(x) < 0$ means the graph is concave down.

Relative Maxima and Minima

Relative maxima and minima occur at the critical numbers ($c.n.$) and can be identified by the 2nd Derivative Test:

$f''(c.n.) > 0$, the point is a relative minimum.

$f''(c.n.) < 0$, the point is a relative maximum.

Points of Inflection

$f''(x) = 0$ and solve for x.

Also where $f''(x)$ does not exist.

Remember inflection points are *points* with coordinates $(x, f(x))$.

Limits at Infinity

Where are the "tails" of the graph pointing? (See Range)

If $x \to \infty^{\pm}$, $f(x) \to \infty^{\pm}$?

$f(x) \to L$ is a horizontal asymptote.